TRIGONOMETRIC IDENTITIES

The six trigonometric functions:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y} = \frac{1}{\tan\theta}$$

Sum or difference of two angles:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

Double angle formulas:

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

Half angle formulas:

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \qquad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

Sum and product formulas:

$$\sin a \cos b = \tfrac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$\cos a \sin b = \tfrac{1}{2}[\sin(a+b) - \sin(a-b)]$$

$$\cos a \cos b = \tfrac{1}{2}[\cos(a+b) + \cos(a-b)]$$

$$\sin a \sin b = \tfrac{1}{2}[\cos(a-b) - \cos(a+b)]$$

$$\sin a + \sin b = 2\sin\left(\tfrac{a+b}{2}\right)\cos\left(\tfrac{a-b}{2}\right)$$

$$\sin a - \sin b = 2\cos\left(\tfrac{a+b}{2}\right)\sin\left(\tfrac{a-b}{2}\right)$$

$$\cos a + \cos b = 2\cos\left(\tfrac{a+b}{2}\right)\cos\left(\tfrac{a-b}{2}\right)$$

$$\cos a - \cos b = -2\sin\left(\tfrac{a+b}{2}\right)\sin\left(\tfrac{a-b}{2}\right)$$

Law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

where A is the angle of a scalene triangle opposite side a.

Radian measure: 8.1 p420

$$1° = \frac{\pi}{180}\text{ radians}$$

$$1\text{ radian} = \frac{180°}{\pi}$$

Reduction formulas:

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$

$$\sin(\theta) = -\sin(\theta-\pi) \qquad \cos(\theta) = -\cos(\theta-\pi)$$

$$\tan(-\theta) = -\tan\theta \qquad \tan(\theta) = \tan(\theta-\pi)$$

$$\mp\sin x = \cos(x \pm \tfrac{\pi}{2}) \qquad \pm\cos x = \sin(x \pm \tfrac{\pi}{2})$$

Complex Numbers:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \tfrac{1}{2}(e^{j\theta} + e^{-j\theta}) \qquad \sin\theta = \tfrac{1}{j2}(e^{j\theta} - e^{-j\theta})$$

TRIGONOMETRIC VALUES FOR COMMON ANGLES

Degrees	Radians	sin q	cos q	tan q	cot q	sec q	csc q
0°	0	0	1	0	Undefined	1	Undefined
30°	π/6	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$	$\sqrt{3}$	$2\sqrt{3}/3$	2
45°	π/4	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	π/3	$\sqrt{3}/2$	1/2	$\sqrt{3}$	$\sqrt{3}/3$	2	$2\sqrt{3}/3$
90°	π/2	1	0	Undefined	0	Undefined	1
120°	2π/3	$\sqrt{3}/2$	-1/2	$-\sqrt{3}$	$-\sqrt{3}/3$	-2	$2\sqrt{3}/3$
135°	3π/4	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
150°	5π/6	1/2	$-\sqrt{3}/2$	$-\sqrt{3}/3$	$-\sqrt{3}$	$-2\sqrt{3}/3$	2
180°	π	0	-1	0	Undefined	-1	Undefined
210°	7π/6	-1/2	$-\sqrt{3}/2$	$\sqrt{3}/3$	$\sqrt{3}$	$-2\sqrt{3}/3$	-2
225°	5π/4	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
240°	4π/3	$-\sqrt{3}/2$	-1/2	$\sqrt{3}$	$\sqrt{3}/3$	-2	$-2\sqrt{3}/3$
270°	3π/2	-1	0	Undefined	0	Undefined	-1
300°	5π/3	$-\sqrt{3}/2$	1/2	$-\sqrt{3}$	$-\sqrt{3}$	2	$-2\sqrt{3}/3$
315°	7π/4	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
330°	11π/6	-1/2	$\sqrt{3}/2$	$-\sqrt{3}/3$	$-\sqrt{3}$	$2\sqrt{3}/3$	-2
360°	2π	0	1	0	Undefined	1	Undefined

Tom Penick tomzap@eden.com www.teicontrols.com/notes 2/20/2000

Expansions for sine, cosine, tangent, cotangent:

$$\sin y = y - \frac{y^3}{6} + \frac{y^5}{5!} - \frac{y^7}{7!} + \cdots$$

$$\cos y = 1 - \frac{y^2}{2} + \frac{y^4}{4!} - \frac{y^6}{6!} + \cdots$$

$$\tan y = y + \frac{y^3}{3} + \frac{2y^5}{15} + \cdots$$

$$\cot y = \frac{1}{y} - \frac{y}{3} - \frac{y^3}{45} - \frac{2y^5}{945} - \cdots$$

Hyperbolic functions:

$$\sinh y = \frac{1}{2}\left(e^{y} - e^{-y}\right) \qquad \sinh \mathrm{j}y = \mathrm{j}\sin y$$

$$\cosh y = \frac{1}{2}\left(e^{y} + e^{-y}\right) \qquad \cosh \mathrm{j}y = \mathrm{j}\cos y$$

$$\tanh \mathrm{j}y = \mathrm{j}\tan y$$

Expansions for hyperbolic functions:

$$\sinh y = y + \frac{y^3}{6} + \cdots$$

$$\cosh y = 1 + \frac{y^2}{2} + \cdots$$

$$\operatorname{sech} y = 1 - \frac{y^2}{2} + \frac{5y^4}{24} - \cdots$$

$$\operatorname{ctnh} y = \frac{1}{y} + \frac{y}{3} - \frac{y^3}{45} + \cdots$$

$$\operatorname{csch} y = \frac{1}{y} - \frac{y}{6} + \frac{7y^3}{360} - \cdots$$

$$\tanh y = y - \frac{y^3}{3} + \frac{2y^5}{15} - \cdots$$